Flow of emulsion through constricted capillary tubes

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Oil recovery by water injection – most used method in the world

The water will flow through the largest pores and leave entrapped oil in the smallest porous space.

Polymer additives: high extensional viscosity is used to control water mobility and improve oil recovery.

Limitations: High cost of polymer additives and physical-chemical interactions.

Use emulsion as blocking agent to direct water to low permeability porous...
Introduction and Motivation

Experiments with alternating water / emulsion injection

Guillen, Alvarado and Carvalho (COBEM, 2007)

Flow Rate = 0.015 ml/min
Emulsion with average drop size 20 µm

Possible mechanism:
Emulsion partially blocks pores already swept by water;

Need to design the emulsion to block the desired area of the reservoirs;

At pore scale, macroscopic model of emulsion flow is not valid;

Need better understanding of the flow.
Introduction and Motivation

Experiments of flow of emulsion through constricted microcapillaries

Experimental set-up

Pressure drop for emulsion with small and large drops

Mobility for different emulsions

Need a flow model to aid on the emulsion design
Emulsion flow in a pore falls into the general class of **microscale flows with interfaces**.

This type of flow appears in many applications in different industries.
Microscale flows with interfaces

Challenges in theoretical analysis

Free surface problem – *boundaries of the flow are not known a priori*;

System of differential equations is highly non-linear;

Capillary effects are extremely important

*Interface configuration needs to be computed very precisely*

\[
P_B - P_A = \frac{2}{R} \sigma = P_C
\]

\[
dF = \sigma ds
\]

Radius of curvature of interface
Microscale flows with interfaces

Development of tools for analysis

Flow Stability

Fundamentals of Coating Process

Two phase flow in porous media

Numerical and Experimental Methods for Free surface and Viscoelastic flows

Applications
Solution Methods

**Interface capturing methods**

Solution on a fixed mesh;
Interface is determined by solving a transport equation;
Level-set, VOF, Mark and Cell, etc...

Simpler
Problems with numerical diffusion near the interface
Inaccurate calculation of interface curvature.

**Interface tracking methods**

Mesh is conformed to the interface;
Equation to find coordinate of nodal points;

More complex;
Mesh equations are usually very stiff;
Very accurate calculation of interface curvature;
Best methods for problems where capillarity in important.
Solution Methods

Interface tracking methods – the mesh is part of the solution

*Fixed point iteration*

Fix the domain
Solve the problem on a fixed mesh
Fix the field variables
Update mesh

Very hard to converge and; When converges, it is at a very slow rate.

*Domain mapping*

Domain is mapped to a reference domain;
Problem is re-written and solved in the reference domain;
Extra equation to describe the mapping.

\[
\begin{bmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y}
\end{bmatrix} = J^{-1}
\begin{bmatrix}
\frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta}
\end{bmatrix}
\]

\[
\nabla_\xi x = J =
\begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}
\end{bmatrix}
\]
Mathematical Formulation

Steady state – straight capillary

Liquid 1

Liquid 2

Conservation equations

\[ 0 = \frac{1}{r} \left( \frac{\partial}{\partial r} (ru_k) + \frac{\partial}{\partial x} (u_k) \right) \]

\[ \rho_k \left( v_k \frac{\partial u_k}{\partial r} + u_k \frac{\partial u_k}{\partial x} \right) = \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{rr} \right) + \frac{\partial}{\partial x} \left( \tau_{xx} \right) \right] - \frac{\partial p_k}{\partial x} \]

\[ \rho_k \left( \frac{\partial v_k}{\partial r} + u_k \frac{\partial v_k}{\partial x} \right) - \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{rt} \right) + \frac{\partial}{\partial x} \left( \tau_{rt} \right) \right] \frac{\partial p_k}{\partial r} \]

Boundary conditions along the interface

\( (u_1 - u_2) = 0 \quad n \cdot (T_1 - T_2) = \frac{\sigma}{R_m} n \quad n \cdot u_1 = 0 \)

Dimensionless parameters:

\[ C_a \equiv \frac{\mu_2 U}{\sigma} \quad \lambda \equiv \frac{\mu_1}{\mu_2} \]
Solution Method – Steady State

Physical domain is mapped into a fixed reference domain \( \mathbf{x} = \mathbf{x}(\xi) \)

Mapping is unknown and arbitrary, but needs to have an inverse and

\[
\det(\nabla_\xi \mathbf{x}) \neq 0 \quad \nabla_\xi \mathbf{x} = \mathbf{J} = \begin{bmatrix}
\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \xi} \\
\frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial r} \\
\frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial \eta}
\end{bmatrix}
\]

boundaries of reference domain have to map boundaries of physical domain
The equations are rewritten in the reference domain;

In the reference domain, the problem is transformed into a regular **Boundary Value Problem**;

An extra field needs to be determined: \( x = x(\xi) \)

The inverse of the mapping is governed by an elliptic differential equation;

\[
\nabla \cdot D \cdot \nabla \xi = 0
\]

Coupling between the mapping equation and the physical problem is through the kinematic boundary condition

\[
- \frac{\partial r}{\partial \eta} \frac{\partial r}{\partial \eta} u + \frac{\partial x}{\partial \eta} \frac{\partial x}{\partial \eta} v = 0
\]
Set of differential equations solved by GALERKIN’S / FINITE ELEMENT METHOD

\[
\begin{bmatrix}
    u \\
    v
\end{bmatrix} = \begin{bmatrix}
    \sum_{i=1}^{n} U_j \phi_j \\
    \sum_{i=1}^{n} V_j \phi_j
\end{bmatrix} \quad \begin{array}{ll}
p = \sum_{i=1}^{m} P_j \chi_j ; & x = \sum_{i=1}^{n} X_j \phi_j ; & r = \sum_{i=1}^{n} R_j \phi_j
\end{array}
\]

- Biquadratic basis functions for \(v\) and \(x\)
- Linear discontinuous functions for \(p\)

The resulting set of non-linear algebraic equations is solved by Newton’s method

Unkowns: \(c = [U_j \ V_j \ P_j \ X_j \ R_j]^T\)

\[
(J) \Delta C^{(k+1)} = -R(C^{(k)})
\]

\[
(C)^{(k+1)} = C^{(k)} + \Delta C^{(k+1)}
\]
Transient Flow – constricted capillary

Set of differential equations solved by GALERKIN’S / FINITE ELEMENT METHOD.

Implicit method for time integration.

Used Newton’s method to solve the set of non-linear algebraic equations at each time step.
Results – steady state

Drop profile as a function of viscosity ratio

The film thickness left on the wall thickens as drop viscosity rises and capillary number rises.
Results – steady state

Streamlines and pressure field near tip of the drop as a function of capillary number

$Ca = 0.1$

$Ca = 0.01$

$Ca = 0.0017$

Viscosity ratio $\lambda = 2$

Steady state solutions are the initial conditions for the transient analysis – flow through a capillary with constriction
Results – Transient flow

Evolution of the drop configuration as it flows through the constriction

Steady state configuration
Mesh evolution as the drop flows through the constriction …

Mesh generation needs to be flexible enough to describe the evolution of the drop configuration
Results – Transient flow

Pressure rises as drop flows through the constriction

Inverse of mobility

Pressure drop of Steady flow

\[ \Delta p \]

Position of drop front

\[ \frac{\Delta P R^2}{\mu Q} \]

\[ X(t)/L \]
At steady state, mobility rises with capillary number thicker film attached to the wall – better lubrication of drop;

At high capillary number, the pressure rise as the drop flows through the capillary throat is small – weak partial blocking of the capillary

At low capillary number, the pressure rise as the drop flows through the capillary throat is large – strong partial blocking of the capillary

Theoretical model was able to describe experimental observations.

Low mobility at low capillary number with large drops. Emulsion is partially blocking the flow

Effect is not observed at high capillary number
Final Remarks

Steady and transient flow of a drop through a capillary was analyzed;

Results show the effect of capillary number (drop velocity) and viscosity ratio on the pressure drop and thickness of the film left on the wall;

Results show the mobility drop as the drop passes by the constriction;

The results explain the experimentally observed partial blocking of capillaries;

Results will help to develop method to use emulsions as pore blocking agents in Enhanced Oil Recovery operations;